PROBLEM 1 - DZONGKHA

ཐཡཱ་འཕྲིན་དང་པོ། - རྫོང་ཁ

Problem 1 (20 points). Here are some numbers in Dzongkha and their numerical values:

1 — **ci** 3 — **sum** 8 — **ge** 12 — **cuɲi** 17 — **cupdyn** 19 — **cygu**

For higher numbers, Dzongkha uses two different systems (referred to here as A and B). Below are some numbers written in both systems, as well as their numerical values:

System A	System B	Value
ke ci da ɲi	**tsaɲi**	22
ke ci da ŋa	**tsaŋa**	25
ke pɟe-da ɲi	**sumcu**	30
ke ci da cyʑi	**soʑi**	34
ke ɲi da dyn	**ʑedyn**	47

System A	System B	Value
ke ko-da sum	**ŋaŋa**	55
ke sum da cuɖu	**døndu**	76
ke ʑi	**gepcu**	80
ke ʑi da gu	**ɟagu**	89
ke ceŋa	**sumɟa**	300

Finally, some equalities are given with left-hand side written in system A and right-hand side written in system B. Some numbers are missing.

	System A		System B
(1)	**cusum + ke pɟe-da ʑi**	=	**ɟasum**
(2)	**ɲiɕu ɲi**	=	**ɲiɕu × ʑipcu**
(3)	**ɲiɕu ci da ke sum da gu**	=	**(ŋapcu × gu) + cygu**
(4)	**ɲiɕu pɟe-da ɲi + ke pɟe-da ɖu**	=	**ŋapɟa + ɲiɟa cutãm**
(5)	**(ɲi × ko) + pɟe**	=	**ɲi**
(6)	**(ɲiɕu ko-da sum × pɟe) + ke pɟe-da sum**	=	**ɖukɟa**
(7)	**ɲiɕu ci da ke cuɖu da cuɖu**	=	**(ɟaʑi × ʑi) + ʑipɟa**
(8)	**ɲi × ɲiɕu ci da ke cutãm da gu**	=	**(____X × ɲiɕu) + copge**
(9)	**____Y + ke ci da ʑi**	=	**ɟaɖu**
(10)	**____Z + ke ko-da ɖu**	=	**dynɟa + sumɟa**

(a) Fill in the blanks X–Z with Dzongkha numbers.

(b) Write with digits the equalities (1–10).

(c) Write in Dzongkha in both systems: 75; 570.

⚠ The Dzongkha language belongs to the Sino-Tibetan family. It is spoken by approx. 171,000 people in Bhutan.

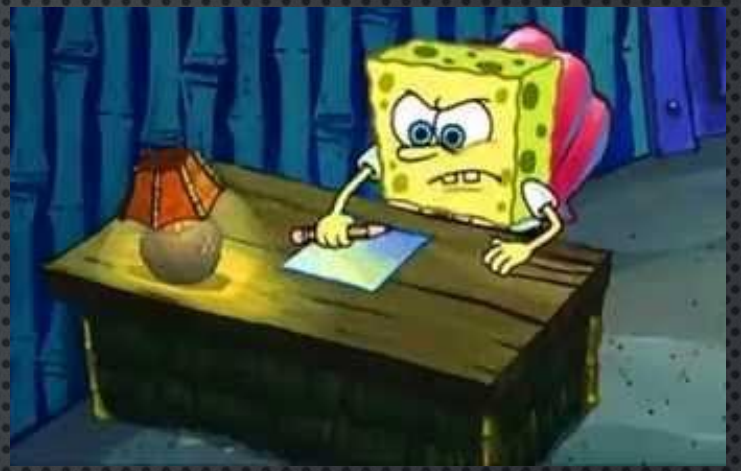
The words are given in a simplified transcription. **ɖ**, **ɟ**, **ɲ**, **ŋ**, **ɕ** and **ʑ** are consonants. **ã**, **ø** and **y** are vowels.

—Vlad A. Neacşu

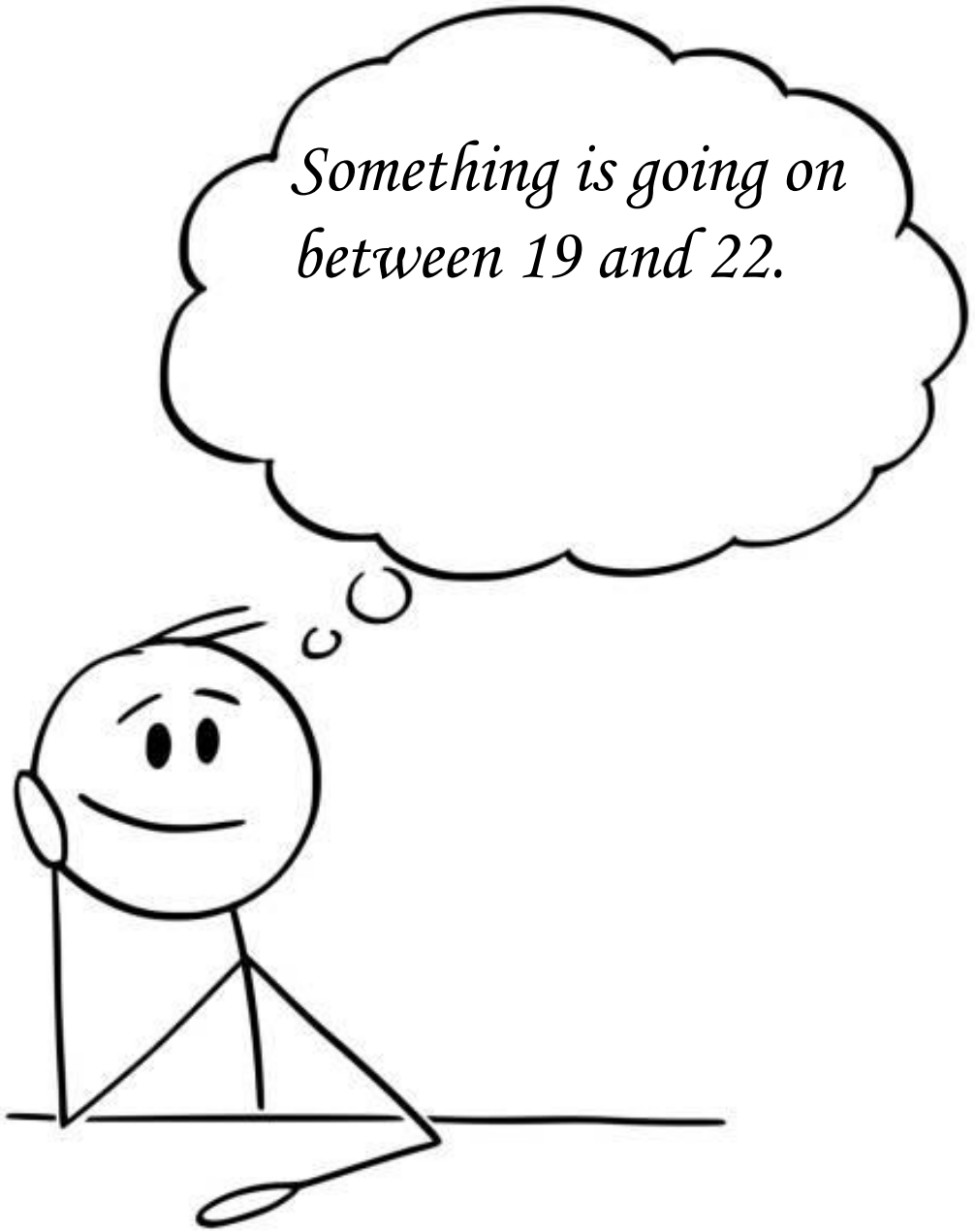

CHINA
BHUTAN
INDIA
BANGLADESH
89°
90°
91°
92°
28°
27°
26°
Punakha
Thimphu
Ha
Paro
Wangdü Phodrang
Tongsa
Bumthang
Lhuntsi
Tashigang
Mongar
Kanglung
Chima Kothi
Chirang
Shemgang
Chhukha
Dagana
Domphu
Pema Gatsel
Sibsoo
Phuntsholing
Gaylegphug
Deothang
Samdrup Jongkhar
Samchi
Shompangkha
0 30 60 mi
0 40 80 km
©1998, Encyclopædia Britannica, Inc.

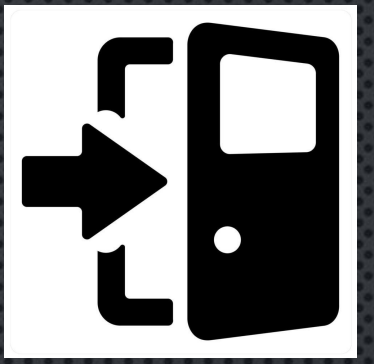

Khampa Tibetan
Dzongkha
Kurtöp
Dzala
Brokkat
Cho-ca-nga-ca-kha
Lakha
Bumthang
Dakpa
'Nyenkha
Chali
Brokpa
'Olekha
Kheng
Tshangla
Gongduk
Lepcha
Lhokpu
Nepali

GRADING TEAM

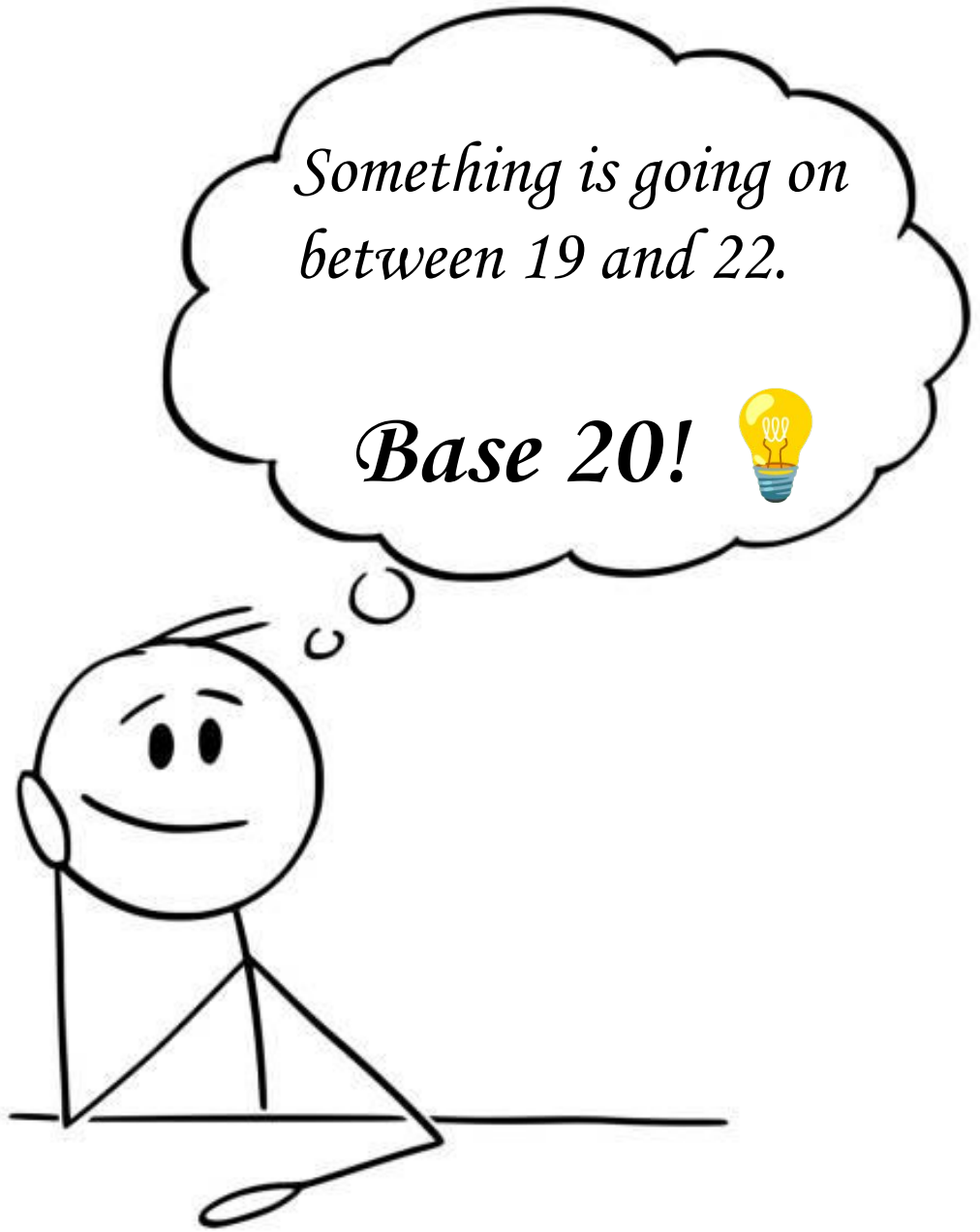


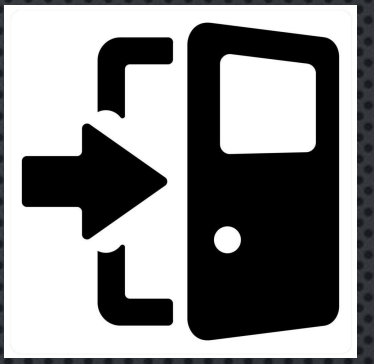
Milena Veneva
Stanislav Gurevich
Pan Tung-Le





19 = **cygu** (in both systems)
22 = **ke ci da ɲi** (system A)
tsaɲi (system B)





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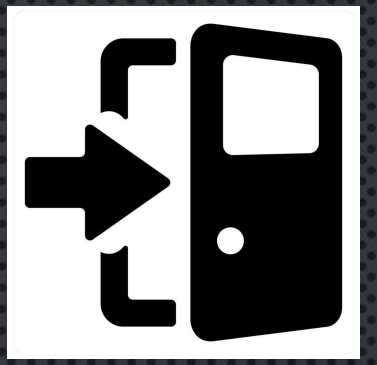
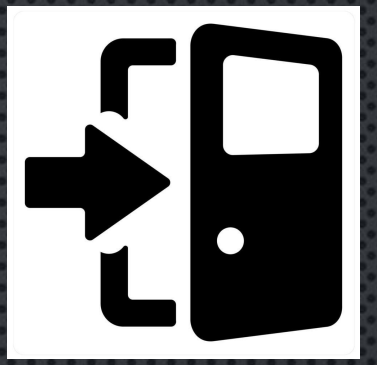
22 = **ke ci da ɲi** (system A)
tsaɲi (system B)



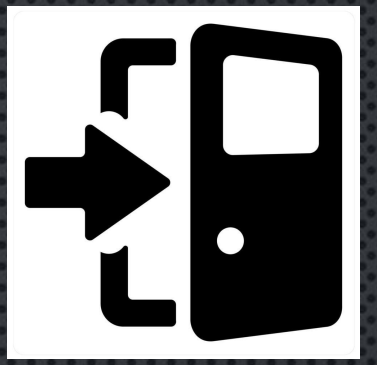
System A:
ke X da Y



System A:
ke X da Y

ke ci(1) da Y: 22, 25, 34
ke ji da Y: 47, 55
ke sum(3) da Y: 76

System A (base 20)
ke X da Y = 20X + Y



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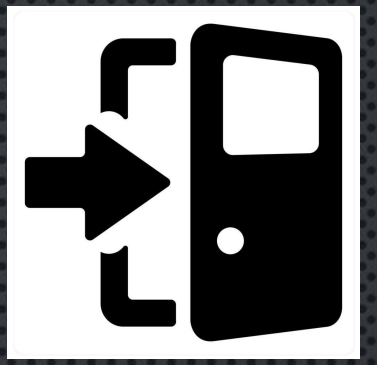
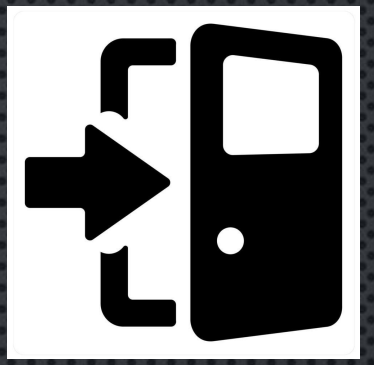


System A:
ke X da Y

ke ci(1) da Y: 22, 25, 34
ke ɲi da Y: 47, 55
ke sum(3) da Y: 76

System B:
ŋaŋa = 55
tsaŋa = 25
tsaɲi = 22

System A (base 20)
ke X da Y = 20X + Y



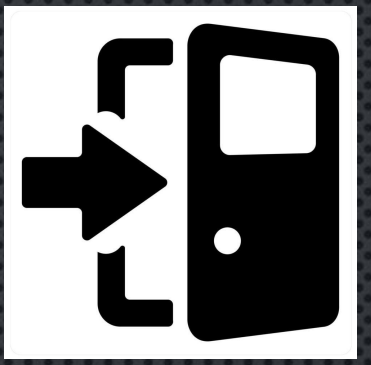
System A:
ke X da Y

ke ci(1) da Y: 22, 25, 34
ke ɲi da Y: 47, 55
ke sum(3) da Y: 76

System B:
ŋaŋa = 55
tsaŋa = 25
tsaɲi = 22

System A (base 20)
ke X da Y = 20X + Y

System B (base 10)
XY = 10X + Y



System A:
ke X da Y

ke ci(1) da Y: 22, 25, 34
ke ɲi da Y: 47, 55
ke sum(3) da Y: 76

System B:
ŋaŋa = 55
tsaŋa = 25
tsaɲi = 22

SYSTEM B (BASE 10)

ŋa-ŋa = 55
tsa-ŋa = 25
tsa-ɲi = 22

	10X	+Y
2	**tsa**	**ɲi**
3		
4		
5	**ŋa**	**ŋa**
6		
7		
8		
9		

SYSTEM B (BASE 10)

ŋa-ŋa = 55
tsa-ŋa = 25
tsa-ɲi = 22

so-ʑi = 34
ʑe-dyn = 47
døn-ɖu = 76
ɟa-gu = 89

	10X	+Y
2	tsa	ɲi
3		
4		
5	ŋa	ŋa
6		
7		
8		
9		

SYSTEM B (BASE 10)

ŋa-ŋa = 55
tsa-ŋa = 25
tsa-ɲi = 22

so-ʑi = 34
ʑe-dyn = 47
døn-ɖu = 76
ɟa-gu = 89

sumcu = 30
gepcu = 80
sumɟa = 300

	10X	+Y
2	tsa	ɲi
3	so	
4	ʑe	ʑi
5	ŋa	ŋa
6		ɖu
7	døn	dyn
8	ɟa	
9		gu

SYSTEM B (BASE 10)

ŋa-ŋa = 55
tsa-ŋa = 25
tsa-ɲi = 22

so-ʑi = 34
ʑe-dyn = 47
døn-ɖu = 76
ɟa-gu = 89

sumcu = 30
gepcu = 80
sumɟa = 300

	10X+	+Y	10X
2	tsa	ɲi	
3	so		sum
4	ʑe	ʑi	
5	ŋa	ŋa	
6		ɖu	
7	døn	dyn	
8	ɟa		gep
9		gu	

– $\beta \times 10 = \textcircled{\beta}$-**cu**, $\beta > 2$;

$\beta \times 10 = \textcircled{\beta}$-**ɕu**, $\beta = 2$

– $\beta \times 10 + \alpha = \textcircled{\textcircled{\beta}}$-$\boxed{\alpha}$

– $\gamma \times 100\,(+\omega) = \textcircled{\gamma}$-**ʄa** ω

	10X+	+Y	10X
2	**ʦa**	**ɲi**	**ɲi**
3	**so**	**sum**	**sum**
4	**ʑe**	**ʑi**	**ʑip**
5	**ŋa**	**ŋa**	**ŋap**
6		**ɖu**	**ɖuk**
7	**døn**	**dyn**	**dyn**
8	**ɟa**	**ge**	**gep**
9		**gu**	

10X + Y = [10X+]—[+Y]

X = 2,9; Y = 1,9

10X = [10X]—**cu**

cu > ɕu for X = 2

X = 2,9

100X (+Y) = [10X]—ʄa (Y)

X = 2,9; Y = 1,99

SYSTEM B (BASE 10)

	10X+	+Y	10X	10+
2	**ʦa**	**ɲi**	**ɲi**	**cu**
3	**so**	**sum**	**sum**	**cu**
4	**ʑe**	**ʑi**	**ʑip**	**cy**
5	**ŋa**	**ŋa**	**ŋap**	**ce**
6		**ɖu**	**ɖuk**	**cu**
7	**døn**	**dyn**	**dyn**	**cup**
8	**ɟa**	**ge**	**gep**	**cop**
9		**gu**		**cy**

$10 + X = [10+]_X — [+X]$

$10X + Y = [10X+] — [+Y]$ X = 2,9; Y = 1,9

$10X = [10X] — \mathbf{cu}$ X = 2,9

cu > ʑu for X = 2

$100X\ (+Y) = [10X] — \mathbf{ɟa}\ (Y)$ X = 2,9; Y = 1,99

SYSTEM A (BASE 20)

ke [+Y]$_X$ da [+Y]$_Y$ = 20X + Y

	10X+	+Y	10X	10+
2	tsa	ɲi	ɲi	cu
3	so	sum	sum	cu
4	ʑe	ʑi	ʑip	cy
5	ŋa	ŋa	ŋap	ce
6		ɖu	ɖuk	cu
7	døn	dyn	dyn	cup
8	ɟa	ge	gep	cop
9		gu		cy

System A	Value
ke ci da ɲi	22
ke ci da ŋa	25
ke pɟe-da ɲi	30
ke ci da cyʑi	34
ke ɲi da dyn	47

System A	Value
ke ko-da sum	55
ke sum da cuɖu	76
ke ʑi	80
ke ʑi da gu	89
ke ceŋa	300

SYSTEM A (BASE 20)

ke $[+Y]_X$ (da $[+Y]_Y$) = 20X (+ Y)

X = 1,19; Y = 1,19

	+Y
2	ɲi
3	sum
4	ʑi
5	ŋa
6	ɖu
7	dyn
8	ge
9	gu

System A	Value
ke ci da ɲi	22
ke ci da ŋa	25
ke pɟe-da ɲi	30
ke ci da cyʑi	34
ke ɲi da dyn	47

System A	Value
ke ko-da sum	55
ke sum da cuɖu	76
ke ʑi	80
ke ʑi da gu	89
ke ceŋa	300

SYSTEM A (BASE 20)

ke $[+Y]_X$ (da $[+Y]_Y$) = 20X (+ Y)

X = 1,19; Y = 1,19

ke pɟe-da X = ???

ke ko-da X = ???

	+Y
2	ɲi
3	sum
4	ʑi
5	ŋa
6	ɖu
7	dyn
8	ge
9	gu

System A	Value
ke ci da ɲi	22
ke ci da ŋa	25
ke pɟe-da ɲi	30
ke ci da cyʑi	34
ke ɲi da dyn	47

System A	Value
ke ko-da sum	55
ke sum da cuɖu	76
ke ʑi	80
ke ʑi da gu	89
ke ceŋa	300

SYSTEM A (BASE 20)

ke $[+Y]_X$ (da $[+Y]_Y$) = 20X (+ Y)

X = 1,19; Y = 1,19

ke pɟe-da X = ???

ke ko-da X = ???

ɲiɕu $[+Y]_X$ (da $[+Y]_Y$) = 400X (+ Y)

X = 1,19; Y = 1,399

ɲiɕu pɟe-da X = ???

ɲiɕu ko-da X = ???

	+Y
2	**ɲi**
3	**sum**
4	**ʑi**
5	**ŋa**
6	**ɖu**
7	**dyn**
8	**ge**
9	**gu**

SYSTEM A (BASE 20)

ke pɟe-da [2] = 30
ke pɟe-da [3] = 50
ke pɟe-da [4] = 70
ke pɟe-da [6] = 110

ke pɟe-da X = 20X – 10

Below are some numbers written in both systems, as well as their numerical values:

System A	System B	Value
ke ci da ɲi	tsaɲi	22
ke ci da ŋa	tsaŋa	25
ke pɟe-da ɲi	sumcu	30
ke ci da cyʑi	soʑi	34
ke ɲi da dyn	ʑedyn	47

System A	System B	Value
ke ko-da sum	ŋaŋa	55
ke sum da cuɖu	døndu	76
ke ʑi	gepcu	80
ke ʑi da gu	ɟagu	89
ke ceŋa	sumɟa	300

Finally, some equalities are given with left-hand side written in system A and right-hand side written in system B. Some numbers are missing.

	System A		System B
(1)	[illegible] + ke pɟe-da ʑi	=	ɟasum
(2)	ɲiɕu ɲi	=	ɲiɕu × ʑipcu
(3)	ɲiɕu ci da ke sum da gu	=	(ŋapcu × gu) + cygu
(4)	[illegible] + ke pɟe-da ɖu	=	ŋapɟa + ɲiɟa cutãm
(5)	(ɲi × ko) + pɟe	=	ɲi
(6)	(ɲiɕu ko-da sum × pɟe) + ke pɟe-da sum	=	ɖukɟa

SYSTEM A (BASE 20)

ke pɟe-da X = 20X – 10

ke ko-da X = 20X – 5

1/2

1/4

ɲiʁu pɟe-da X = 400X – 200

ɲiʁu ko-da X = 400X – 100

$$(2 \times \text{ko}) + \text{pɟe} = 2$$

SYSTEM A (BASE 20)

ke ɲɟe-da X = 20(X-1) + 10

ke ko-da X = 20(X-1) + 15

½

¾

ɲiʁu ɲɟe-da X = 400(X-1) + 200

ɲiʁu ko-da X = 400(X-1) + 300

$$(2 \times \tfrac{3}{4}) + \tfrac{1}{2} = 2$$

System A

System B

- Base 20
- Overcounting for ½ and ¾
- **ɲiɐ̯u = 400**

- Base 10
- Five different forms for each digit
- **ɲiɐ̯u = 20**

		1	2	3	4	5	6	7	8	9
$\boxed{k}$	k	ci	ɲi	sum	ʑi	ŋa	ɖu	dyn	ge	gu
	$10(+k)$	cu-	cu-	cu-	cy-	ce-	cu-	cup-	cop-	cy-
$\textcircled{k}$	$k \times 10$	cutãm	ɲi-	sum-	ʑip-	ŋap-	ɖuk-	dyn-	gep-	(gup-)
$\textcircled{\textcircled{k}}$	$k \times 10(+\ldots)$		tsa-	so-	ʑe-	ŋa-	(re-)	døn-	ɟa-	(go-)

- System A:
  - $\beta \times 20\ (+\alpha)$ = **ke** $\boxed{\beta}$ (**da** $\boxed{\alpha}$)
    - $\beta \times 20 + 10 \rightarrow$ **ke pɟe-da** $\boxed{\beta+1}$
    - $\beta \times 20 + 15 \rightarrow$ **ke ko-da** $\boxed{\beta+1}$
  - $\gamma \times 400\ (+\beta \times 20 + \alpha)$ = **ɲiɕu** $\boxed{\gamma}$ (**da ke** $\boxed{\beta}$ **da** $\boxed{\alpha}$)
    - $\gamma \times 400 + 200 \rightarrow$ **ɲiɕu pɟe-da** $\boxed{\gamma+1}$
    - $\gamma \times 400 + 300 \rightarrow$ **ɲiɕu ko-da** $\boxed{\gamma+1}$

- System B:
  - $\beta \times 10$ = $\textcircled{\beta}$**-cu**, $\beta > 2$;
    $\beta \times 10$ = $\textcircled{\beta}$**-ɕu**, $\beta = 2$
  - $\beta \times 10 + \alpha$ = $\textcircled{\textcircled{\beta}}$**-** $\boxed{\alpha}$
  - $\gamma \times 100\ (+\omega)$ = $\textcircled{\gamma}$**-ɟa** ω

(b)

(1) $13 + 70 = 83$

(2) $800 = 20 \times 40$

(3) $469 = 50 \times 9 + 19$

(4) $600 + 110 = 500 + 210$

(5) $2 \times \frac{3}{4} + \frac{1}{2} = 2$

(6) $1100 \times \frac{1}{2} + 50 = 600$

(7) $736 = 84 \times 4 + 400$

(8) $2 \times 609 = 60_X \times 20 + 18$

(9) $62_Y + 24 = 86$

(10) $885_Z + 115 = 700 + 300$

(a)

[X] = 60 = **ɖukcu**

[Y] = 62 = **ke sum da ɲi**

[Z] = 885 = **ɲiɕu ɲi da ke ʑi da ŋa**

(c)

- 75 = **ke ko-da ʑi** = **dønŋa**
- 570 = **ɲiɕu ci da ke pɟe-da gu** = **ŋapɟa dyncu**

TRIVIA

- System A (base 20) is used in the every day life, while System B (base 10) is more formal and it is calqued from Classical Tibetan.
- System A (base 20) is still widely used in counting amounts of houses, dogs, boxes, and crates.
- When stating an amount or a price, the decade is included in the number.

45 (**ʑeŋa**) will become **ʑipcu ʑeŋa** (40 45).

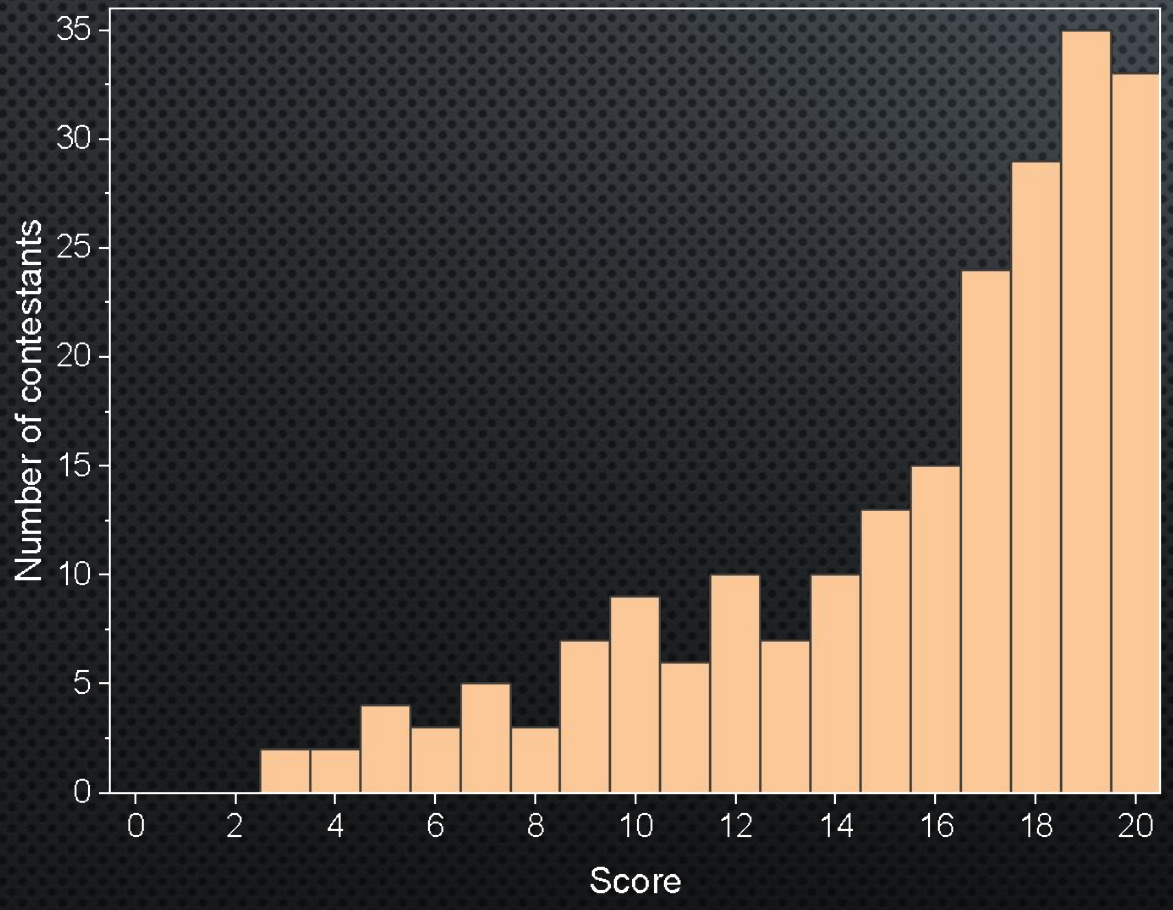
- Separate set of numerals for 21-29 used for the days of the months.

YOUR OPINION

EASIEST PROBLEM: 81/206 (39.3 %)

HARDEST PROBLEM: 11/206 (5.3 %)

OUR OPINION



SUBMISSIONS: 217/227 (95.6 %)

AVERAGE SCORE: 15.28 / 20

MAX SCORES: 11


THE
END?

BEST SOLUTION(S)

BEST SOLUTION(S)

TANANON KINTHORN

THAILAND

BEST SOLUTIONS

BEST SOLUTIONS

Ekaterina Churkina

Belka